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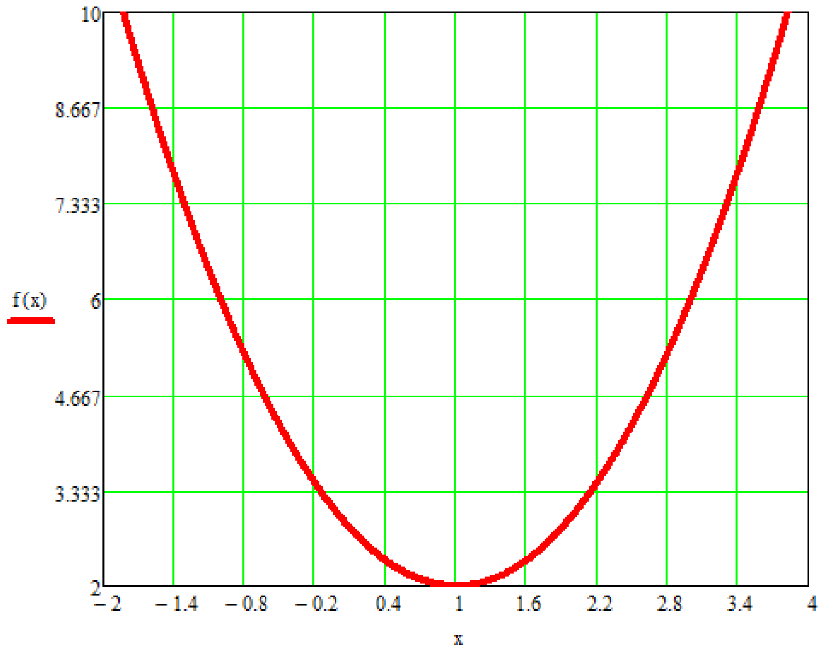
AL 54 Să se determine imaginea funcției $f : R \to R$, $f(x) = x^2 - 2x + 3$

$$V\left(-\frac{b}{2a};-\frac{\Delta}{4a}\right) \Rightarrow V\left(\frac{2}{2}; f(1)\right) \Rightarrow V(1;2)$$

$$f(1) = 1 - 2 + 3 = 2$$

$$\Rightarrow \operatorname{Im} f = [2;+\infty)$$

b



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AL 61 Se dau funcțiile $f: R \to R$, $f(x) = x^2 - x$ și $g: [0;+\infty) \to R$, $g(x) = \sqrt{x}$

Dacă notăm cu $h: [0;+\infty) \to R$ funcția compusă $f \circ g$, să se determine h(x)

$$h(x) = (f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = x - \sqrt{x}$$

d

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AL 70 Fie numărul $x_0 = \sqrt[3]{\sqrt{5}+2} - \sqrt[3]{\sqrt{5}-2}.$

Care din următoarele afirmații este adevărată?

$$x_0^3 = \sqrt{5}+2-3(\sqrt[3]{\sqrt{5}+2}\cdot\sqrt[3]{\sqrt{5}-2})(\sqrt[3]{\sqrt{5}+2}-\sqrt[3]{\sqrt{5}-2})-\sqrt{5}+2$$

$$x_0^3 = 4-3x_0$$

$$x_0^3+3x_0-4=0$$

$$x_0^3-x_0+4x_0-4=0$$

$$x_0(x_0^2-1)+4(x_0-1)=0$$

$$(x_0-1)[x_0^2+x_0+4]=0$$

$$x_0 = 1$$

d

$$\Delta = 1-16$$

$$\Delta = -15$$

$$x_0 \notin R$$

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AL 71 Fie mulțimea $M = \{x \in R : |x| \geq 1, \sqrt[2013]{x+\sqrt{x^2-1}} + \sqrt[2013]{x-\sqrt{x^2-1}} = 2\}$.

Dacă $S = \sum_{x \in M} x^2$ atunci

$$\sqrt[2013]{x+\sqrt{x^2-1}} + \sqrt[2013]{x-\sqrt{x^2-1}} = 2$$

$$(\sqrt[2013]{x+\sqrt{x^2-1}})^2 + \sqrt[2013]{x^2-x^2+1} = 2\sqrt[2013]{x+\sqrt{x^2-1}}$$

$$(\sqrt[2013]{x+\sqrt{x^2-1}})^2 + 1 = 2\sqrt[2013]{x+\sqrt{x^2-1}}$$

$$\sqrt[2013]{x+\sqrt{x^2-1}} = t$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$$t = 1$$

$$\sqrt[2013]{x+\sqrt{x^2-1}} = 1$$

$$x + \sqrt{x^2-1} = 1$$

$$\sqrt{x^2-1} = 1 - x$$

$$x = 1$$

$$S = \sum_{x \in M} x^2 = 1$$

a

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AL 72 Să se determine valoarea expresiei $E=\sqrt[3]{7+5\sqrt{2}}+\sqrt[3]{7-5\sqrt{2}}$

$$E=\sqrt[3]{7+5\sqrt{2}}+\sqrt[3]{7-5\sqrt{2}}$$

$$E^3=7+5\sqrt{2}+3\sqrt[3]{7+5\sqrt{2}}\cdot\sqrt[3]{7-5\sqrt{2}}\left(\sqrt[3]{7+5\sqrt{2}}+\sqrt[3]{7-5\sqrt{2}}\right)+7-5\sqrt{2}$$

$$E^3=7+5\sqrt{2}+3\cdot(-1)\cdot E+7-5\sqrt{2}$$

$$E^3=14-3E$$

$$E^3+3E-14=0$$

$$E^3-2E^2+2E^2-4E+7E-14=0$$

$$E^2(E-2)+2E(E-2)+7(E-2)=0$$

$$(E-2)(E^2+2E+7)=0$$

$$E=2$$

$$\Delta=4-28=-24$$

$$E\notin R$$

c

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AL 73 Să se rezolve ecuația: $\log_2(x+1)=\log_4(x^2-x+4)$

$$\begin{cases} x+1>0 \\ x^2-x+4>0 \end{cases} \Rightarrow \begin{cases} x>-1 \\ x\in R \end{cases} \Rightarrow x\in(-1;+\infty)$$

$$\log_2(x+1)=\log_4(x^2-x+4)$$

$$\log_2(x+1)=\frac{1}{2}\log_2(x^2-x+4)$$

$$\log_2(x+1)=\log_2\sqrt{x^2-x+4}$$

$$x+1=\sqrt{x^2-x+4}$$

$$x^2+2x+1=x^2-x+4$$

$$3x=3$$

$$x=1$$

b

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AL 74

Să se precizeze mulțimea soluțiilor ecuației $3^x \cdot x - 3^x \log_2 3 - 5x + 5\log_2 3 = 0$

$$3^x \cdot x - 3^x \log_2 3 - 5x + 5\log_2 3 = 0$$

$$3^x (x - \log_2 3) - 5(x - \log_2 3) = 0$$

$$(3^x - 5)(x - \log_2 3) = 0$$

$$3^x - 5 = 0$$

$$3^x = 5$$

$$\log_3 3^x = \log_3 5$$

$$x = \log_3 5$$

$$x - \log_2 3 = 0$$

$$x = \log_2 3$$

$$\{\log_2 3; \log_3 5\}$$

b

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AL 75 Să se rezolve în mulțimea numerelor reale: $\begin{cases} \log_3(x^2+2x)=1 \\ 2\cdot 3^y + 9^y = 15 \end{cases}$

$\log_3(x^2+2x)=1$

$x^2+2x>0 \Rightarrow x\in(-\infty;-2)\cup(0;+\infty)$

$x^2+2x=3^1$

$x^2+2x-3=0$

$x_1=1$

$x_2=-3$

$2\cdot 3^y+9^y=15$

$3^y=t$

$t^2+2t-15=0$

$t_1=3\Rightarrow 3^y=3\Rightarrow y=1$

$t_2=-5$ nu este soluție

b

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AL 76 Să se determine toate valorile parametrului $m \in R$ pentru care inecuația $3^{2x} - 3^{x+1}m + 2m^2 < 0$ nu are soluții reale.

$$3^{2x} - 3^{x+1}m + 2m^2 \geq 0 \forall x \in R$$

Fie: $3^x = t$

$$t^2 - 3mt + 2m^2 \geq 0 \forall t \in (0;+\infty)$$

t	0				$+\infty$
$t^2 - 3mt + 2m^2$	|	+	0	+	+

$$\begin{cases} a > 0 \\ \Delta \leq 0 \end{cases} \Rightarrow 9m^2 - 8m^2 \leq 0 \Rightarrow m^2 \leq 0 \Rightarrow m = 0$$

e

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AL 77

Să se rezolve în mulțimea numerelor reale inecuația:

$$\log_x(x^2 - x + 2) > 2\log_x(x+1)$$

$$\begin{cases} x^2 - x + 2 > 0 \\ x > 0; x \neq 1 \\ x + 1 > 0 \end{cases} \Rightarrow x > 0; x \neq 1 \Rightarrow x \in (0;1) \cup (1;+\infty)$$

$x \in (0;1)$

$$\log_x(x^2 - x + 2) > 2\log_x(x+1)$$

$$\log_x(x^2 - x + 2) > \log_x(x+1)^2$$

$$x^2 - x + 2 < x^2 + 2x + 1$$

$$-3x < -1$$

$$x > \frac{1}{3}$$

$$x \in (\frac{1}{3};1)$$

$x \in (1;+\infty)$

$$\log_x(x^2 - x + 2) > 2\log_x(x+1)$$

$$\log_x(x^2 - x + 2) > \log_x(x+1)^2$$

$$x^2 - x + 2 > x^2 + 2x + 1$$

$$-3x > -1$$

$x < \frac{1}{3}$ nu este soluție

a

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AL 79 Să se determine $m \in R$ astfel încât inecuația $(\frac{1}{4})^x - m(\frac{1}{2})^x + 1 > 0$ să fie adevărată pentru orice x<0.

Fie: $(\frac{1}{2})^x = t$

Dacă

$x \in (-\infty;0) \Rightarrow t \in (1;+\infty)$

$$(\frac{1}{4})^x - m(\frac{1}{2})^x + 1 > 0$$

$$t^2 - mt + 1 > 0; t \in (1;+\infty)$$

t	1				$+\infty$
$t^2 - mt + 1 > 0$	2-m	+	+	+	+

$$\begin{cases} 2 - m > 0 \\ -\frac{b}{a} < 1 \end{cases} \Rightarrow \begin{cases} m < 2 \\ \frac{m}{1} < 1 \end{cases} \Rightarrow \begin{cases} m < 2 \\ m < 1 \end{cases} \Rightarrow m < 1$$

e

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AL 80 Stiind că $a=\log_2 3,$ să se calculeze în funcție de a, expresiile

$$E=\log_2 9-\log_4 3-\log_2 36+\log_3 144$$

$$F=\log_3 9+2\log_4 27-\log_3 36+\log_{12} 144$$

$$E=\log_2 9-\log_4 3-\log_2 36+\log_3 144=\log_2\frac{9}{36}-\frac{1}{2}\log_2 3+\log_3(2^4\cdot 3^2)=$$

$$=\log_2\frac{1}{4}-\frac{1}{2}a+4\log_3 2+2\log_3 2=-2-\frac{1}{2}a+\frac{4}{a}+2=\frac{8-a^2}{2a}$$

$$F=\log_3 9+2\log_4 27-\log_3 36+\log_{12} 144=\log_3\frac{9}{36}+2\cdot\frac{1}{2}\log_2 3^3+\log_{12} 12^2=$$

$$=\log_3\frac{1}{4}+3\log_2 3+2=\log_3 1-\log_3 4+3a+2=-\log_3 2^2+3a+2=-2\log_3 2+3a+2=$$

$$=-\frac{2}{a}+3a+2$$

d

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AL 81

Valorile numărului natural nenul n pentru care următoarea inegalitate: $\sqrt{a}\cdot\sqrt[6]{a}\cdot\sqrt[12]{a}\cdot\ldots\cdot\sqrt[n(n+1)]{a}<a^n, a>0, a\neq 1,$ este adevărată, sunt:

$$\sqrt{a}\cdot\sqrt[6]{a}\cdot\sqrt[12]{a}\cdot\ldots\cdot\sqrt[n(n+1)]{a}<a^n$$

$$a^{\frac{1}{2}}\cdot a^{\frac{1}{6}}\cdot a^{\frac{1}{12}}\cdot\ldots\cdot a^{\frac{1}{n(n+1)}}<a^n$$

$$a^{\sum_{k=1}^{n}\frac{1}{k(k+1)}}<a^n$$

$$a^{\frac{n}{n+1}}<a^n$$

$a\in(0;1)$	$a\in(1;+\infty)$
$\dfrac{n}{n+1}>n$	$\dfrac{n}{n+1}<n$
$\dfrac{1}{n+1}>1$	$\dfrac{1}{n+1}<1$
$1>n+1$	$1<n+1$
$n<0$	$n>0$

d

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AL 82

Fie suma $$S=\frac{1}{\sum_{k=1}^{n}\log_2 k}+\frac{1}{\sum_{k=1}^{n}\log_3 k}+\ldots+\frac{1}{\sum_{k=1}^{n}\log_n k};n\in N^*;n\geq 2$$

$$\sum_{k=1}^{n}\log_2 k=\log_2 1+\log_2 2+\ldots+\log_2 n=\log_2 1\cdot 2\cdot\ldots\cdot n=\log_2 n!$$

$$S=\frac{1}{\log_2 n!}+\frac{1}{\log_3 n!}+\ldots+\frac{1}{\log_n n!}=\log_{n!}2+\log_{n!}3+\ldots+\log_{n!}n=$$

$$=\log_{n!}2\cdot 3\cdot\ldots\cdot n=\log_{n!}n!=1$$

e

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AL 83

În ce interval se află rădăcina strict pozitivă a ecuației

$$(x^2+9)^{\frac{1}{\log_x(x^2+9)}} = \sqrt[3]{-x^2+6x}$$

$$(x^2+9)^{\frac{1}{\log_x(x^2+9)}} = \sqrt[3]{-x^2+6x} \qquad x \in (0;1) \cup (1;+\infty)$$

$$(x^2+9)^{\log_{x^2+9} x} = \sqrt[3]{-x^2+6x}$$

$$x = \sqrt[3]{-x^2+6x}$$

$$x^3 = -x^2+6x$$

$$x^3+x^2-6x=0$$

$$x(x^2+x-6)=0$$

$$x_1 = 0 \qquad x_2 = 2 \in (1;6) \qquad x_3 = -3$$

c

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AL 84

Care este valoarea produsului P, al rădăcinilor ecuației

$$x^2 + x(4^x - 14) + 4^{x+1} - 72 = 0$$

$$x^2 + x(4^x - 14) + 4^{x+1} - 72 = 0$$

$$x^2 + x \cdot 4^x - 14x + 4^{x+1} - 72 = 0$$

$$x^2 + 4x - 18x - 72 + x \cdot 4^x + 4 \cdot 4^x = 0$$

$$x(x+4) - 18(x+4) + 4^x(x+4) = 0$$

$$(x+4)(x - 18 + 4^x) = 0$$

$$x + 4 = 0$$

$$x_1 = -4$$

$$x - 18 + 4^x = 0$$

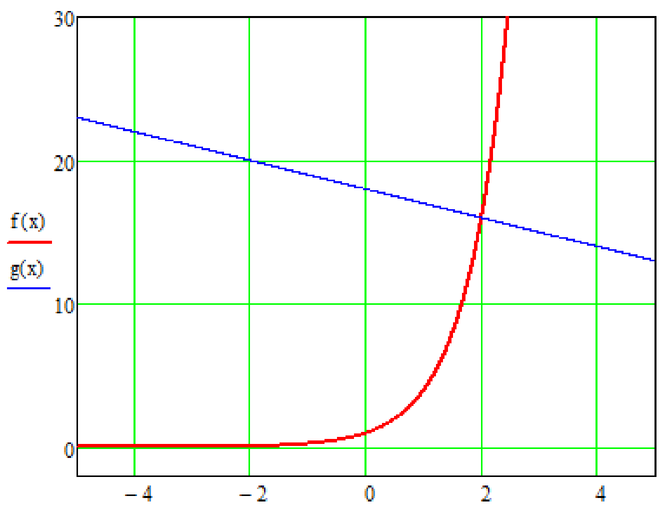
$$4^x = 18 - x$$

$$x_2 = 2$$

$$P = x_1 \cdot x_2 = -8$$

a

$f(x) := 4^x$ $g(x) := 18 - x$



$f(2) = 16$

$\lim_{x \to -\infty} f(x) \to 0$

$\lim_{x \to \infty} f(x) \to \infty$

$g(2) = 16$

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AL 85

Care este valoarea S, a rădăcinilor ecuației:

$$\left(\sqrt{9+4\sqrt{5}}\right)^{\frac{x}{3}}+\left(\sqrt{9-4\sqrt{5}}\right)^{\frac{x}{3}}=18$$

$$\left(\sqrt{9+4\sqrt{5}}\right)^{\frac{x}{3}}+\left(\sqrt{9-4\sqrt{5}}\right)^{\frac{x}{3}}=18$$

$$\left(\sqrt{9+4\sqrt{5}}\right)^{\frac{2x}{3}}+1=18\left(\sqrt{9+4\sqrt{5}}\right)^{\frac{x}{3}}$$

Fie: $\left(\sqrt{9+4\sqrt{5}}\right)^{\frac{x}{3}}=t$ $\quad t^2-18t+1=0$

$$t_{1,2}=\frac{18\pm\sqrt{324-4}}{2}=\frac{18\pm8\sqrt{5}}{2}=9\pm4\sqrt{5}$$

$$\left(\sqrt{9+4\sqrt{5}}\right)^{\frac{x}{3}}=9+4\sqrt{5}$$

$$(9+4\sqrt{5})^{\frac{x}{6}}=9+4\sqrt{5}$$

$$\frac{x}{6}=1$$

$$x_1=6$$

$$\left(\sqrt{9+4\sqrt{5}}\right)^{\frac{x}{3}}=9-4\sqrt{5}$$

$$(9+4\sqrt{5})^{\frac{x}{6}}=\frac{1}{9+4\sqrt{5}}$$

$$\frac{x}{6}=-1$$

$$x_2=-6$$

$$S=x_1+x_2=6-6=0$$

d

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AL 86 Fie $x_1;x_2;\ldots.x_n$ numerele reale astfel încât $x_1;x_2;\ldots.x_n \in (0;1)$ sau $x_1;x_2;\ldots.x_n \in (1;+\infty)$. Precizați care este valoarea minimă a expresiei:

$$E = \log_{x_1}(x_2x_3\cdot\ldots\cdot x_n) + \log_{x_2}(x_1x_3\cdot\ldots\cdot x_n) + \ldots + \log_{x_n}(x_1x_2\cdot\ldots\cdot x_{n-1})$$

$$\log_{x_1} x_2 + \log_{x_2} x_1 \geq 2$$

$$\log_{x_1} x_2 + \frac{1}{\log_{x_1} x_2} \geq 2$$

$$(\log_{x_1} x_2)^2 - 2\log_{x_1} x_2 + 1 \geq 0$$

$$(\log_{x_1} x_2 - 1)^2 \geq 0$$

x_1x_2

x_1x_3 x_2x_3

……………………………………

x_1x_n x_2x_n ……………… $x_{n-1}x_n$

====================================

2(n-1) 2(n-2) ……………… 2*1

$$E = [\log_{x_1} x_2 + \log_{x_1} x_3 + \ldots + \log_{x_1} x_n] + [\log_{x_2} x_1 + \log_{x_2} x_3 + \ldots + \log_{x_{21}} x_n] + \ldots +$$

$$+ [\log_{x_n} x_1 + \log_{x_n} x_2 + \ldots + \log_{x_n} x_{n-1}] \geq 2(n-1) + 2(n-2) + \ldots 2\cdot 1 =$$

$$= 2[(n-1) + (n-2) + \ldots + 1] = 2\cdot\frac{1+n-1}{2}\cdot(n-1) = n(n-1)$$

f

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AL 87 Să se determine domeniul maxim de definiție al funcției

$$f(x)=\sqrt[4]{\log_{x^2}\left(\left|\frac{x-1}{x+1}\right|-1\right)}$$

$I)\ x\in(-\infty;-1)$

$$\begin{cases}\log_{x^2}\left(\left|\frac{x-1}{x+1}\right|-1\right)\ge 0\\ \left|\frac{x-1}{x+1}\right|-1>0\end{cases}$$

$$\begin{cases}\left|\frac{x-1}{x+1}\right|-1\ge 1\\ \left|\frac{x-1}{x+1}\right|>1\end{cases}$$

$$\begin{cases}\frac{x-1}{x+1}>2\\ \frac{x-1}{x+1}>1\end{cases}$$

$$x-1\le 2x+2$$

$$-x\le 3$$

$$x\ge -3$$

$$\boxed{x\in[-3;-1)}$$

$II)\ x\in(-1;0)$

$$\begin{cases}\log_{x^2}\left(\left|\frac{x-1}{x+1}\right|-1\right)\ge 0\\ \left|\frac{x-1}{x+1}\right|-1>0\end{cases}$$

$$\begin{cases}\left|\frac{x-1}{x+1}\right|-1\le 1\\ \left|\frac{x-1}{x+1}\right|-1>0\end{cases}$$

$$\begin{cases}\frac{1-x}{x+1}\le 2\\ \frac{1-x}{x+1}>1\end{cases}$$

$$\begin{cases}1-x\le 2x+2\\ 1-x>x+1\end{cases}$$

$$\begin{cases}-3x\le 1\\ -2x>0\end{cases}$$

$$\begin{cases}x\ge -\frac{1}{3}\\ x<0\end{cases}$$

$$\boxed{x\in[-\frac{1}{3};0)}$$

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$x \in (0;1)$

$$\begin{cases} \log_{x^2}\left(\left|\frac{x-1}{x+1}\right| - 1\right) \geq 0 \\ \left|\frac{x-1}{x+1}\right| - 1 > 0 \end{cases}$$

$$\begin{cases} \left|\frac{x-1}{x+1}\right| - 1 \leq 1 \\ \left|\frac{x-1}{x+1}\right| > 1 \end{cases}$$

$$\begin{cases} \frac{1-x}{x+1} \leq 2 \\ \frac{1-x}{x+1} > 1 \end{cases}$$

$$\begin{cases} 1 - x \leq 2x + 2 \\ 1 - x > x + 1 \end{cases}$$

$$\begin{cases} -3x \leq 1 \\ -2x > 1 \end{cases}$$

$$\begin{cases} x \geq -\frac{1}{3} \\ x < -\frac{1}{2} \end{cases}$$

nu are soluții

$x \in (1;+\infty)$

$$\begin{cases} \log_{x^2}\left(\left|\frac{x-1}{x+1}\right| - 1\right) \geq 0 \\ \left|\frac{x-1}{x+1}\right| - 1 > 0 \end{cases}$$

$$\begin{cases} \left|\frac{x-1}{x+1}\right| - 1 \geq 1 \\ \left|\frac{x-1}{x+1}\right| > 1 \end{cases}$$

$$\begin{cases} \frac{x-1}{x+1} \geq 2 \\ \frac{x-1}{x+1} > 1 \end{cases}$$

$x - 1 \geq 2x + 2$

$-x \geq 3$

$x \leq -3$

nu are soluții

Soluția generală: $x \in [-3;-1) \cup [-\frac{1}{3};0)$

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AL 89 Se consideră mulțimea

$M=\{(x,y,z):2^{x+z}+3^y=91;2^x+3^y\cdot 2^z=436;2^x\cdot 3^y+2^z=124\}\subset ZxZxZ$ și

$S=\sum\limits_{(x,y,z)\in M}(|x|+|y|+|z|)$. Atunci:

$$2^{x+z}+3^y=91$$

$$2^{x+z}=91-3^y$$

$$\begin{cases} y=3 \\ 2^{x+z}=64 \end{cases}$$

$$\begin{cases} y=3 \\ x+z=6 \end{cases}$$

$$\begin{cases} 2^x+27\cdot 2^z=436 \\ 2^x\cdot 27+2^z=124 \end{cases} \Rightarrow \begin{cases} 2^{6-z}+27\cdot 2^z=436 \\ 2^{6-z}\cdot 27+2^z=124 \end{cases} \Rightarrow \begin{cases} \dfrac{2^6}{2^z}+27\cdot 2^z=436 \\ \dfrac{2^6}{2^z}\cdot 27+2^z=124 \end{cases} \Rightarrow$$

Fie: $2^z=t$

$$\begin{cases} \dfrac{64}{t}+27\cdot t=436 \\ \dfrac{64}{t}\cdot 27+t=124 \end{cases} \Rightarrow \begin{cases} 64+27t^2-436t=0 \\ 1728+t^2-124t=0 \end{cases} \Rightarrow \begin{cases} 27t^2-436t+64=0 \\ t^2-124t+1728=0 \end{cases} \Rightarrow$$

$$\Rightarrow t=16 \Rightarrow \begin{cases} 2^z=16 \\ x=6-z \end{cases} \Rightarrow \begin{cases} z=4 \\ x=2 \end{cases} \Rightarrow S=|2|+|3|+|4|=9$$

a

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AL 90

Fie mulțimea

$$A=\{x\in R:\lg 2+\sqrt{\lg(\lg x)}=\lg(10+\lg x)\}$$ atunci:

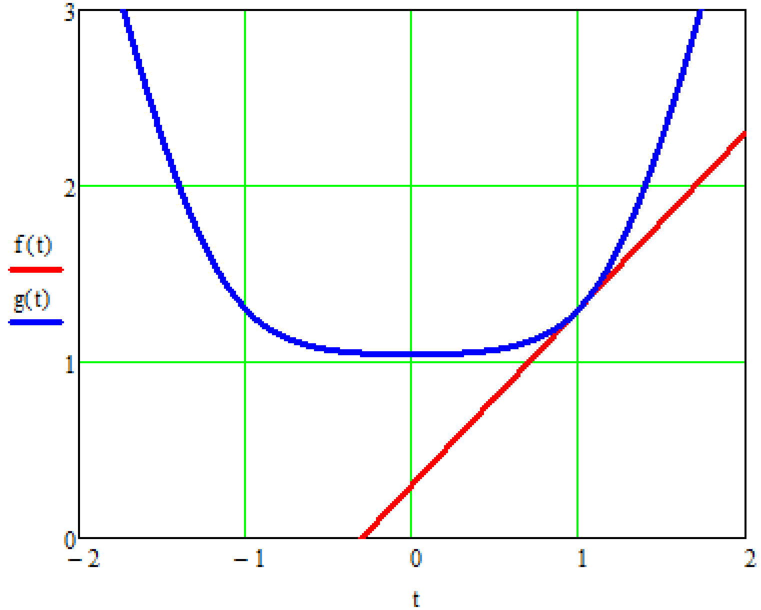
$$\lg 2+\sqrt{\lg(\lg x)}=\lg(10+\lg x)$$

$$\lg 2+t=\lg(10+10^{t^2})$$

Fie $\sqrt{\lg(\lg x)}=t$

$$\lg(\lg x)=t^2$$

$$\lg x=10^{t^2}$$



$f(1) = 1.301$

$g(1) = 1.301$

$$\Rightarrow t=1\Rightarrow\sqrt{\lg(\lg x)}=1\Rightarrow\lg(\lg x)=1\Rightarrow\lg x=10\Rightarrow x=10^{10}\in(10^8;10^{12})$$

e

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AL 94

Să se determine toate perechile (x, y), cu $x, y \in N^*$, astfel încât:

$$C_x^y = C_y^x$$
$$(x+y)! < 1000$$

$$\begin{cases} C_x^y = C_y^x \\ (x+y)! < 1000 \end{cases} \Rightarrow \begin{cases} x \geq y \\ y \geq x \\ (x+y)! < 1000 \end{cases} \Rightarrow \begin{cases} x = y \\ (2x)! = 1000 \end{cases} \Rightarrow \begin{cases} x \in \{1;2;3\} \\ y \in \{1;2;3\} \end{cases}$$

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AL 95

Fie

$$M=\sqrt{C_{2n+1}^{1}\cdot C_{2n+1}^{2}\cdot C_{2n+1}^{3}\cdot\ldots\cdot C_{2n+1}^{2n}},n\in N^{*}$$ atunci:

$$M=\sqrt{C_{2n+1}^{1}\cdot C_{2n+1}^{2}\cdot C_{2n+1}^{3}\cdot\ldots\cdot C_{2n+1}^{2n}}=\sqrt{(C_{2n+1}^{1})^{2}\cdot(C_{2n+1}^{2})^{2}\cdot(C_{2n+1}^{3})^{2}\cdot\ldots\cdot(C_{2n+1}^{n})^{2}}=$$
$$=C_{2n+1}^{1}\cdot C_{2n+1}^{2}\cdot C_{2n+1}^{3}\cdot\ldots\cdot C_{2n+1}^{n}\in N^{*}$$

d

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AL 96 Fie șirul $x_n=(1-\frac{1}{3})(1-\frac{1}{6})\cdot\ldots(1-\frac{1}{C_{n+1}^2}), n\in N^*\setminus\{1\}$

Definim mulțimea $M:=\{n\in N^*\setminus\{1\}:\frac{1}{2}\le x_n\le\frac{2}{3}\}$

$$x_n=(1-\frac{1}{3})(1-\frac{1}{6})\cdot\ldots(1-\frac{1}{C_{n+1}^2})=\prod_{k=2}^{n}(1-\frac{1}{\frac{(k+1)\cdot k}{2}})=\prod_{k=2}^{n}(1-\frac{2}{(k+1)\cdot k})=$$

$$=\prod_{k=2}^{n}\frac{k^2+k-2}{(k+1)\cdot k}=\prod_{k=2}^{n}\frac{(k-1)(k+2)}{k\cdot(k+1)}=\frac{1\cdot 4}{2\cdot 3}\cdot\frac{2\cdot 5}{3\cdot 4}\cdot\frac{3\cdot 6}{4\cdot 5}\cdot\ldots\cdot\frac{(n-2)(n+1)}{(n-1)n}\cdot\frac{(n-1)(n+2)}{n(n+1)}=$$

$$=\frac{n+2}{3n}$$

$$\frac{1}{2}\le x_n\le\frac{2}{3}\Rightarrow\frac{1}{2}\le\frac{n+2}{3n}\le\frac{2}{3}\Rightarrow 3n\le 2n+4\le 4n\Rightarrow\begin{cases}3n\le 2n+4\\2n+4\le 4n\end{cases}\Rightarrow n\in\{2;3;4\}\Rightarrow$$

$$\Rightarrow M=\{2;3;4\}$$

c

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Care este probabilitatea p, respectiv q, ca alegând una din rădăcinile ecuației $(x+2)(x^2-x-1)=0$ aceasta să fie reală, respectiv întreagă.

$$(x+2)(x^2-x-1)=0$$

$$x_1=-2\in Z$$

$$x_{2,3}=\frac{1\pm\sqrt{5}}{2}\in R$$

$$\Rightarrow \begin{cases} p=\dfrac{3}{3}=1 \\ q=\dfrac{1}{3} \end{cases}$$

b